

## অনুশীলনী-১৩

### অনির্দিষ্ট যোজিতফল (Indefinite Integral)

অতিসংক্ষিপ্ত প্রশ্নঃ

১.  $\int \frac{dx}{a^2+x^2} =$  কত?

সমাধানঃ  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$  (Ans)

২.  $\int \frac{dx}{2x} =$  কত?

সমাধানঃ  $\int \frac{dx}{2x} = \frac{1}{2} \log x + c$  (Ans)

৩.  $\int \frac{dx}{\sqrt{x}} =$  কত?

সমাধানঃ  $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + c$  (Ans)

৪.  $\int \frac{dx}{2\sqrt{x}} =$  কত?

সমাধানঃ  $\int \frac{dx}{2\sqrt{x}} = \frac{1}{2} 2\sqrt{x} + c = \sqrt{x} + c$  (Ans)

৬.  $\int \frac{dx}{9+x^2} =$  কত?

সমাধানঃ  $\int \frac{dx}{9+x^2} = \int \frac{dx}{3^2+x^2} = \frac{1}{3} \tan^{-1} \frac{x}{3} + c$  (Ans)

৭.  $\int \frac{dx}{16+x^2} =$  কত?

সমাধানঃ  $\int \frac{dx}{16+x^2} = \int \frac{dx}{4^2+x^2} = \frac{1}{4} \tan^{-1} \frac{x}{4} + c$  (Ans)

৮.  $\int \frac{dx}{\sqrt{a^2-x^2}} =$  কত?

সমাধানঃ  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  (Ans)

৯.  $\int \frac{dx}{\sqrt{4-x^2}} =$  কত?

সমাধানঃ  $\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{dx}{\sqrt{2^2-x^2}} = \sin^{-1} \frac{x}{2} + c$  (Ans)

১০.  $\int uvdx =$  কত?

সমাধানঃ  $\int uvdx = u \int vdx - \int \left\{ \frac{du}{dx} \int vdx \right\} dx$  (Ans)

## সংক্ষিপ্ত ও রচনামূলক প্রশ্নঃ

১.  $\int \text{Sin}x\text{Sin}2x dx$  এর মান নির্ণয় কর।

$$\text{সমাধানঃ} \int \text{Sin}x\text{Sin}2x dx = \frac{1}{2} \int 2\text{Sin}2x\text{Sin}x dx = \frac{1}{2} \int \{\text{Cos}(2x - x) - \text{Cos}(3x + x)\} dx$$

$$= \frac{1}{2} \int (\text{Cos}x - \text{Cos}3x) dx = \frac{1}{2} \left( \text{Sin}x - \frac{\text{Sin}3x}{3} \right) + c = \frac{1}{6} (3\text{Sin}x - \text{Sin}3x) + c \quad (\text{Ans})$$

২.  $\int \text{Sin}5x\text{Sin}3x dx$  এর মান নির্ণয় কর।

$$\text{সমাধানঃ} \int \text{Sin}5x\text{Sin}3x dx = \frac{1}{2} \int 2\text{Sin}5x\text{Sin}3x dx$$

$$= \frac{1}{2} \int \{\text{Cos}(5x - 3x) - \text{Cos}(5x + 3x)\} dx$$

$$= \frac{1}{2} \int (\text{Cos}2x - \text{Cos}8x) dx = \frac{1}{2} \left( \frac{\text{Sin}2x}{2} - \frac{\text{Sin}8x}{8} \right) + c$$

$$= \frac{1}{16} (4\text{Sin}2x - \text{Sin}8x) + c \quad (\text{Ans})$$

৩.  $\int \sin^2 x dx$  এর মান নির্ণয় কর।

$$\begin{aligned}\text{সমাধানঃ} \int \sin^2 x dx &= \frac{1}{2} \cdot \int 2\sin^2 x dx = \frac{1}{2} \cdot \int (1 - \cos 2x) dx \\ &= \frac{1}{2} \left( x - \frac{\sin 2x}{2} \right) + c = \frac{1}{4} (2x - \sin 2x) + c \text{ (Ans)}\end{aligned}$$

৪.  $\int \cos^2 x dx$  এর মান নির্ণয় কর।

$$\begin{aligned}\text{সমাধানঃ} \int \cos^2 x dx &= \frac{1}{2} \cdot \int 2\cos^2 x dx = \frac{1}{2} \int (1 + \cos 2x) dx \\ &= \frac{1}{2} \left( x + \frac{\sin 2x}{2} \right) + c = \frac{1}{4} (2x + \sin 2x) + c \text{ (Ans)}\end{aligned}$$

৫.  $\int \cos^4 x dx$  এর মান নির্ণয় কর।

$$\begin{aligned}\text{সমাধানঃ} \cos^4 x &= \frac{1}{4} \cdot (2\cos^2 x)^2 = \frac{1}{4} \cdot (1 + \cos 2x)^2 = \frac{1}{4} (1 + 2\cos 2x + \cos^2 2x) \\ &= \frac{1}{4} \cdot \left[ 1 + 2\cos 2x + \frac{1}{2} \cdot (2\cos^2 2x) \right] = \frac{1}{4} \cdot \left[ 1 + 2\cos 2x + \frac{1}{2} (1 + \cos 4x) \right] \\ &= \frac{1}{4} + \frac{\cos 2x}{2} + \frac{1}{8} + \frac{\cos 4x}{8} = \frac{3}{8} + \frac{\cos 2x}{2} + \frac{\cos 4x}{8}\end{aligned}$$

$$\begin{aligned}\text{এখন, } \int \cos^4 x dx &= \int \left( \frac{3}{8} + \frac{\cos 2x}{2} + \frac{\cos 4x}{8} \right) dx = \frac{3x}{8} + \frac{\sin 2x}{2 \cdot 2} + \frac{\sin 4x}{8 \cdot 4} + c \\ &= \frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c \text{ (Ans)}\end{aligned}$$

৬.  $\int \sin^4 x dx$  এর মান নির্ণয় কর।

$$\begin{aligned}\text{সমাধানঃ } \sin^4 x &= \frac{1}{4} \cdot (2\sin^2 x)^2 = \frac{1}{4} \cdot (1 - \cos 2x)^2 = \frac{1}{4} (1 - 2\cos 2x + \cos^2 2x) \\ &= \frac{1}{4} \left[ 1 - 2\cos 2x + \frac{1}{2} \cdot (2\cos^2 2x) \right] = \frac{1}{4} \left[ 1 - 2\cos 2x + \frac{1}{2} (1 + \cos 4x) \right] \\ &= \frac{1}{4} - \frac{\cos 2x}{2} + \frac{1}{8} + \frac{\cos 4x}{8} = \frac{3}{8} - \frac{\cos 2x}{2} + \frac{\cos 4x}{8}\end{aligned}$$

$$\begin{aligned}\text{এখন, } \int \sin^4 x dx &= \int \left( \frac{3}{8} - \frac{\cos 2x}{2} + \frac{\cos 4x}{8} \right) dx = \frac{3x}{8} - \frac{\sin 2x}{2 \cdot 2} + \frac{\sin 4x}{8 \cdot 4} + c \\ &= \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c \quad (\text{Ans})\end{aligned}$$

৭.  $\int \tan^3 x \sec^2 x dx$  এর মান নির্ণয় কর।

সমাধানঃ দেওয়া আছে,  $\int \tan^3 x \sec^2 x dx$

$$\begin{aligned}&= \int z^3 dz \\ &= \frac{z^4}{4} + c \\ &= \frac{1}{4} (\tan x)^4 + c \quad (\text{Ans})\end{aligned}$$

ধরি,  $z = \tan x$

$$\text{বা, } \frac{dz}{dx} = \frac{d}{dx} (\tan x)$$

$$\text{বা, } \frac{dz}{dx} = \sec^2 x$$

$$\therefore dz = \sec^2 x dx$$

৮.  $\int e^{\tan^{-1}x} \cdot \frac{1}{1+x^2} dx$  এর মান নির্ণয় কর।

সমাধানঃ দেওয়া আছে,  $\int e^{\tan^{-1}x} \cdot \frac{1}{1+x^2} dx$

$$= \int e^z dz$$

$$= e^z + c$$

$$= e^{\tan^{-1}x} + c \text{ (Ans)}$$

৯.  $\int \frac{1}{x(1+\log x)} dx$  এর মান নির্ণয় কর।

সমাধানঃ দেওয়া আছে,  $\int \frac{1}{x(1+\log x)} dx$

$$= \int \frac{dz}{z}$$

$$= \log z + c$$

$$= \log(1 + \log x) + c \text{ (Ans)}$$

ধরি,  $z = \tan^{-1}x$

$$\text{বা, } \frac{dz}{dx} = \frac{d}{dx} (\tan^{-1}x)$$

$$\text{বা, } \frac{dz}{dx} = \frac{1}{1+x^2}$$

$$\therefore dz = \frac{dx}{1+x^2}$$

ধরি,  $z = 1 + \log x$

$$\text{বা, } \frac{dz}{dx} = \frac{d}{dx} (\log x)$$

$$\text{বা, } \frac{dz}{dx} = \frac{1}{x}$$

$$\therefore dz = \frac{dx}{x}$$

১০.  $\int (1 + \text{Cos}x)^5 \text{Sin}x dx$  এর মান নির্ণয় কর।

সমাধানঃ দেওয়া আছে,  $\int (1 + \text{Cos}x)^5 \text{Sin}x dx$

$$= - \int z^5 dz$$

$$= - \frac{z^6}{6} + c$$

$$= -\frac{1}{6} (1 + \text{Cos}x)^6 + c \quad (\text{Ans})$$

১১.  $\int \frac{\tan x}{\log(\text{Cos}x)} dx$  এর মান নির্ণয় কর।

সমাধানঃ দেওয়া আছে,  $\int \frac{\tan x}{\log(\text{cos}x)} dx$

$$= - \int \frac{dz}{z}$$

$$= -\log z + c$$

$$= -\log(\log \text{cos}x) + c \quad (\text{Ans})$$

ধরি,  $z = 1 + \text{Cos}x$

$$\text{বা, } \frac{dz}{dx} = \frac{d}{dx} (1 + \text{Cos}x)$$

$$\text{বা, } \frac{dz}{dx} = 0 - \text{Sin}x$$

$$\therefore -dz = \text{Sin}x dx$$

ধরি,  $z = \log(\text{Cos}x)$

$$\text{বা, } \frac{dz}{dx} = \frac{d}{dx} \{\log(\text{Cos}x)\}$$

$$\text{বা, } \frac{dz}{dx} = \frac{1}{\text{Cos}x} \cdot \frac{d}{dx} (\text{Cos}x)$$

$$\text{বা, } \frac{dz}{dx} = \frac{1}{\text{Cos}x} \cdot (-\text{Sin}x)$$

$$\text{বা, } \frac{dz}{dx} = -\tan x$$

$$\therefore -dz = \tan x dx$$

১২.  $\int \tan^{-1}x dx$  এর মান নির্ণয় কর।

সমাধানঃ  $\int \tan^{-1}x \cdot 1 \cdot dx$

$$= \tan^{-1}x \int 1 \cdot dx - \int \left\{ \frac{d}{dx} (\tan^{-1}x) \int 1 \cdot dx \right\} dx = \tan^{-1}x \cdot x - \int \left\{ \frac{1}{1+x^2} \cdot x \right\} dx$$

$$= x \cdot \tan^{-1}x - \frac{1}{2} \int \frac{2x}{1+x^2} dx = x \cdot \tan^{-1}x - \frac{1}{2} \cdot \log(1+x^2) + c \text{ (Ans)}$$

১৩.  $\int x \tan^{-1}x dx$  এর মান নির্ণয় কর।

সমাধানঃ  $\int x \cdot \tan^{-1}x dx$

$$= \tan^{-1}x \int x \cdot dx - \int \left\{ \frac{d}{dx} (\tan^{-1}x) \int x \cdot dx \right\} dx = \tan^{-1}x \cdot \frac{x^2}{2} - \int \left\{ \frac{1}{1+x^2} \cdot \frac{x^2}{2} \right\} dx$$

$$= \frac{x^2}{2} \cdot \tan^{-1}x - \frac{1}{2} \cdot \int \left\{ \frac{(1+x^2)-1}{1+x^2} \right\} dx = \frac{x^2}{2} \cdot \tan^{-1}x - \frac{1}{2} \cdot \int \left\{ 1 - \frac{1}{1+x^2} \right\} dx$$

$$= \frac{x^2}{2} \tan^{-1}x - \frac{1}{2} (x - \tan^{-1}x) + c \text{ (Ans)}$$