

অনুশীলনী-৭

ফাংশনের সীমা (Limit of a Function)

প্রয়োজনীয় সূত্রাবলীঃ

$$১. \lim_{\theta \rightarrow 0} \frac{\sin\theta}{\theta} = 1$$

$$২. \lim_{\theta \rightarrow 0} \frac{\theta}{\sin\theta} = 1$$

$$৩. \lim_{\theta \rightarrow 0} \frac{\tan\theta}{\theta} = 1$$

$$৪. \lim_{\theta \rightarrow 0} \frac{\theta}{\tan\theta} = 1$$

$$৫. \lim_{\theta \rightarrow 0} \cos\theta = 1$$

$$৬. \lim_{\theta \rightarrow 0} \sec\theta = 1$$

$$৭. \lim_{\theta \rightarrow 0} \sin\theta = 0$$

$$৮. \lim_{\theta \rightarrow 0} \tan\theta = 0$$

$$৯. \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$১০. \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

অতিসংক্ষিপ্ত প্রশ্নঃ

১.মান নির্ণয় করঃ $\lim_{x \rightarrow 2} \frac{x^3 - 2^3}{x - 2}$

সমাধানঃ $\lim_{x \rightarrow 2} \frac{x^3 - 2^3}{x - 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{x - 2}$

$$= \lim_{x \rightarrow 2} x^2 + 2x + 4 = 2^2 + 2 \cdot 2 + 4 = 12 \quad (\text{Ans})$$

২.মান নির্ণয় করঃ $\lim_{x \rightarrow 0} \frac{x^3 - 2^3}{x - 2}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{x^3 - 2^3}{x - 2} = \lim_{x \rightarrow 0} \frac{(x-2)(x^2 + 2x + 4)}{x - 2}$

$$= \lim_{x \rightarrow 0} x^2 + 2x + 4 = 0^2 + 2 \cdot 0 + 4 = 4(\text{Ans})$$

৩.মান নির্ণয় করঃ $\lim_{x \rightarrow \infty} \frac{x}{2x+1}$

সমাধানঃ $\lim_{x \rightarrow \infty} \frac{x}{2x+1} = \lim_{x \rightarrow \infty} \frac{x}{x(2+\frac{1}{x})} = \lim_{x \rightarrow \infty} \frac{1}{2+\frac{1}{x}} = \frac{1}{2+\frac{1}{\infty}} = \frac{1}{2+0} = \frac{1}{2}$ (Ans)

৪.মান নির্ণয় করঃ $\lim_{x \rightarrow \infty} \frac{2x+1}{5x-8}$

সমাধানঃ $\lim_{x \rightarrow \infty} \frac{2x+1}{5x-8} = \lim_{x \rightarrow \infty} \frac{x(2+\frac{1}{x})}{x(5-\frac{8}{x})} = \lim_{x \rightarrow \infty} \frac{2+\frac{1}{x}}{5-\frac{8}{x}} = \frac{2+\frac{1}{\infty}}{5-\frac{8}{\infty}} = \frac{2+0}{5-0} = \frac{2}{5}$ (Ans)

৫. $\lim_{x \rightarrow 0} \frac{x^n - a^n}{x-a}$ এর মান কত ?

সমাধানঃ $\lim_{x \rightarrow 0} \frac{x^n - a^n}{x-a} = na^{n-1}$ (Ans)

সংক্ষিপ্ত প্রশ্নঃ

১.মান নির্ণয় করঃ $\lim_{x \rightarrow 0} \frac{x}{2-\sqrt{4-x}}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{x}{2-\sqrt{4-x}} = \lim_{x \rightarrow 0} \frac{x(2+\sqrt{4-x})}{(2-\sqrt{4-x})(2+\sqrt{4-x})} = \lim_{x \rightarrow 0} \frac{x(2+\sqrt{4-x})}{4-4+x}$

$$= \lim_{x \rightarrow 0} \frac{x(2+\sqrt{4-x})}{x} = \lim_{x \rightarrow 0} 2 + \sqrt{4-x} = 2 + \sqrt{4-0} = 2 + 2 = 4 \text{ (Ans)}$$

২.মান নির্ণয় করঃ $\lim_{x \rightarrow 0} \frac{\sqrt{1-x}-1}{x}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{\sqrt{1-x}-1}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{1-x}-1)(\sqrt{1-x}+1)}{x(\sqrt{1-x}+1)} = \lim_{x \rightarrow 0} \frac{1-x-1}{x(\sqrt{1-x}+1)}$

$$= \lim_{x \rightarrow 0} \frac{-x}{x(\sqrt{1-x}+1)} = \lim_{x \rightarrow 0} \frac{-1}{(\sqrt{1-x}+1)} = \frac{-1}{(\sqrt{1-0}+1)} = \frac{-1}{2} \text{ (Ans)}$$

৩.মান নির্ণয় কর : $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{1+x-1-x}{x(\sqrt{1+x}+\sqrt{1-x})}$

$= \lim_{x \rightarrow 0} \frac{2x}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{2}{(\sqrt{1+x}+\sqrt{1-x})} = \frac{2}{(\sqrt{1+0}+\sqrt{1-0})} = \frac{2}{2} = 1$ (Ans)

৪.মান নির্ণয় কর : $\lim_{x \rightarrow 0} \frac{a-\sqrt{a^2-x^2}}{x^2}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{a-\sqrt{a^2-x^2}}{x^2} = \lim_{x \rightarrow 0} \frac{(a-\sqrt{a^2-x^2})(a+\sqrt{a^2-x^2})}{x^2(a+\sqrt{a^2-x^2})} = \lim_{x \rightarrow 0} \frac{(a^2-a^2+x^2)}{x^2(a+\sqrt{a^2-x^2})}$

$= \lim_{x \rightarrow 0} \frac{x^2}{x^2(a+\sqrt{a^2-x^2})} = \lim_{x \rightarrow 0} \frac{1}{(a+\sqrt{a^2-x^2})} = \frac{1}{(a+\sqrt{a^2-0})} = \frac{1}{2a}$ (Ans)

৫.মান নির্ণয় কর : $\lim_{x \rightarrow 0} \frac{1-\cos 2x}{x}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{1-\cos 2x}{x} = \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \cdot 2x$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^2 \cdot \lim_{x \rightarrow 0} 2x = 1 \cdot (2 \cdot 0) = 0 \quad (\text{Ans})$$

৬.মান নির্ণয় কর : $\lim_{x \rightarrow 0} \frac{1-\cos x}{x^2}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{1-\cos x}{x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 \frac{x}{2}}{x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{\frac{x^2}{4}} \cdot \frac{1}{4} \cdot 2$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2} \quad (\text{Ans})$$

৭.মান নির্ণয় কর : $\lim_{x \rightarrow 0} \frac{1-\cos 7x}{x^2}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{1-\cos 7x}{3x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 \frac{7x}{2}}{3x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 \frac{7x}{2}}{\frac{49x^2}{4}} \cdot \frac{49}{4} \cdot \frac{2}{3}$

$= \lim_{x \rightarrow 0} \left(\frac{\sin \frac{7x}{2}}{\frac{7x}{2}} \right)^2 \cdot \frac{49}{6} = 1 \cdot \frac{49}{6} = \frac{49}{6} \text{ (Ans)}$

৮.মান নির্ণয় কর : $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{\tan x(1 - \cos x)}{x^3}$

$= \lim_{x \rightarrow 0} \frac{\tan x}{x} \cdot \lim_{x \rightarrow 0} \frac{(1 - \cos x)}{x^2} = 1 \cdot \lim_{x \rightarrow 0} \frac{2\sin^2 \frac{x}{2}}{x^2}$

$= \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{\frac{x^2}{4}} \cdot \frac{1}{4} \cdot 2 = \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2} \text{ (Ans)}$

৯.মান নির্ণয় কর : $\lim_{x \rightarrow 0} \frac{\tan 2x - \sin 2x}{x^3}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{\tan 2x - \sin 2x}{x^3} = \lim_{x \rightarrow 0} \frac{\tan 2x(1 - \cos 2x)}{x^3}$

$$= \lim_{x \rightarrow 0} \frac{\tan 2x}{x} \lim_{x \rightarrow 0} \frac{(1 - \cos 2x)}{x^2} = \lim_{x \rightarrow 0} \frac{\tan 2x}{2x} \cdot 2 \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2}$$

$$= 1 \cdot 2 \cdot \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = 2 \cdot 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^2 = 4 \cdot 1 = 4 \text{ (Ans)}$$

১০.মান নির্ণয় কর : $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$

সমাধানঃ $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\tan x(1 - \cos x)}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{\sin x(1 - \cos x)}{\cos x \cdot \sin^3 x}$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x)}{\cos x \cdot \sin^2 x}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{(1-\cos x)}{\cos x(1-\cos^2 x)} = \lim_{x \rightarrow 0} \frac{(1-\cos x)}{\cos x(1-\cos x)(1+\cos x)} = \lim_{x \rightarrow 0} \frac{1}{\cos x(1+\cos x)} \\
&= \lim_{x \rightarrow 0} \frac{1}{\cos x} \cdot \lim_{x \rightarrow 0} \frac{1}{1+\cos x} = \frac{1}{\cos 0} \cdot \frac{1}{1+\cos 0} = 1 \cdot \frac{1}{2} = \frac{1}{2} \text{ (Ans)}
\end{aligned}$$

১১. মান নির্ণয় কর :

$$\lim_{x \rightarrow 0} \frac{\cos 7x - \cos 9x}{\cos 3x - \cos 5x}$$

সমাধানঃ

$$\begin{aligned}
\lim_{x \rightarrow 0} \frac{\cos 7x - \cos 9x}{\cos 3x - \cos 5x} &= \lim_{x \rightarrow 0} \frac{2\sin\left(\frac{7x+9x}{2}\right)\sin\left(\frac{9x-7x}{2}\right)}{2\sin\left(\frac{3x+5x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)} = \lim_{x \rightarrow 0} \frac{\sin 8x \cdot \sin x}{\sin 4x \cdot \sin x} \\
&= \lim_{x \rightarrow 0} \frac{\sin 8x}{\sin 4x} = \lim_{x \rightarrow 0} \frac{\sin 2(4x)}{\sin 4x} = \lim_{x \rightarrow 0} \frac{2\sin 4x \cos 4x}{\sin 4x} \\
&= \lim_{x \rightarrow 0} 2\cos 4x = 2\cos 0 = 2 \text{ (Ans)}
\end{aligned}$$

১২.মান নির্ণয় কর : $\lim_{x \rightarrow a} \frac{x^{5/2} - a^{5/2}}{\sqrt{x} - \sqrt{a}}$

সমাধান : $\lim_{x \rightarrow a} \frac{x^{5/2} - a^{5/2}}{\sqrt{x} - \sqrt{a}} = \lim_{x \rightarrow a} \frac{(\sqrt{x})^5 - (\sqrt{a})^5}{\sqrt{x} - \sqrt{a}}$

মনেকরি, $\sqrt{x} = y$ এবং $\sqrt{a} = b$ \therefore যখন, $x \rightarrow a$ তখন, $y \rightarrow b$

$$= \lim_{y \rightarrow b} \frac{y^5 - b^5}{y - b} = 5b^4 = 5(\sqrt{a})^4 = 5a^2 \text{ (Ans)}$$

Thanks Everybody